

Three

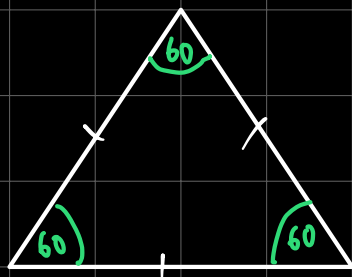
Angles

TRIGONOMETRY

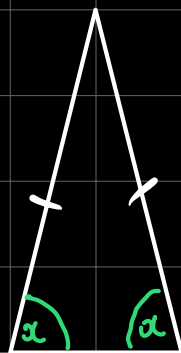
(20-25 Marks)

→ MENSURATION
(AREA/VOLUMES)

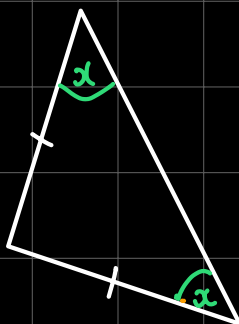
TRIANGLES



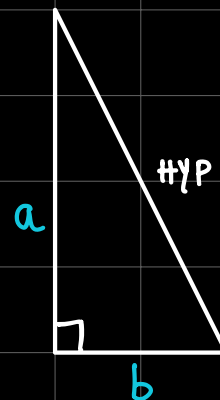
EQUILATERAL



ISOSCELES

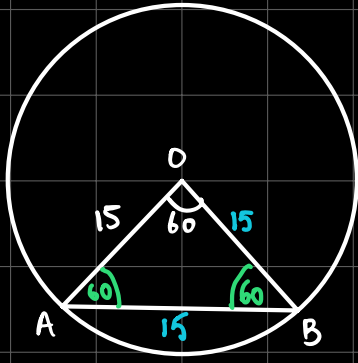


IF ANY ONE ANGLE OF AN
ISOSCELES TRIANGLE IS 60,
IT AUTOMATICALLY BECOMES
AN EQUILATERAL TRIANGLE.



RIGHT ANGLED

'a' and 'b'
are called the
legs of right
angle.



State value of length AB. (1 mark)

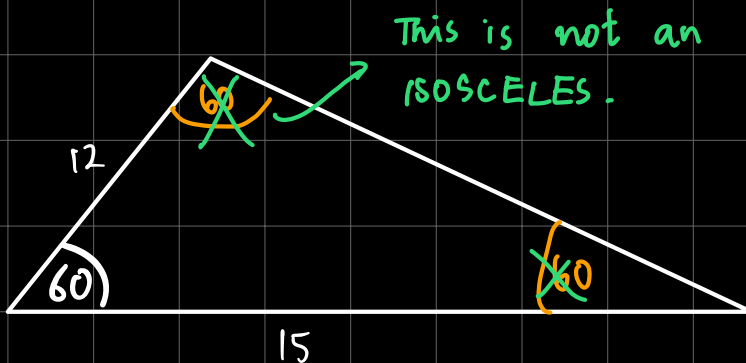
$$AB = 15$$

THOUGHT PROCESS:

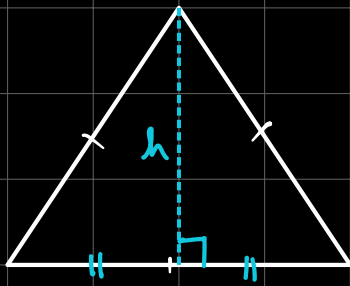
- 1) $OA = OB = 15$ (radius of circle)
- 2) Now we know $\triangle OAB$ is isosceles
- 3) Angle at O is 60

IF ANY ONE ANGLE OF AN
ISOSCELES TRIANGLE IS 60,
IT AUTOMATICALLY BECOMES
AN EQUILATERAL TRIANGLE.

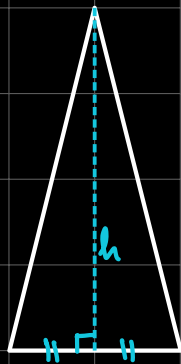
- 4) Hence all angles and sides are equal.



If I draw a height in a triangle, will it cut the base in two equal halves?

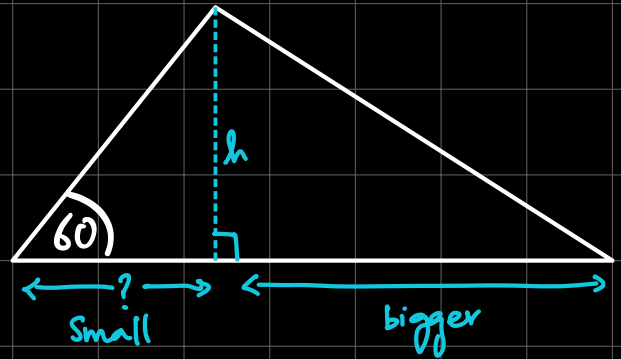


EQUILATERAL



ISOSCELES

In case of equilateral and isosceles triangles, it cuts in two equal halves.



SCALENE

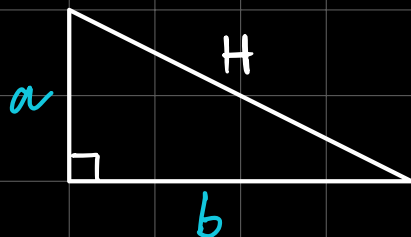
All sides are different

All angles are different.

RIGHT ANGLED TRIANGLE

PYTHAGORAS THEOREM

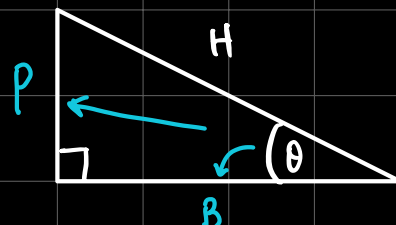
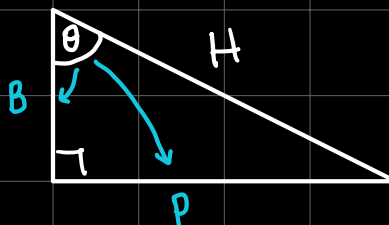
No other angles than 90° are involved $\leftarrow$ given to be found.



$$H^2 = a^2 + b^2$$

TRIGONOMETRIC RATIOS

One angle other than 90° is involved $\leftarrow$ given to be found.



$$\sin \theta = \frac{P}{H}$$

$$\sin \theta = \frac{O}{H}$$

$$\cos \theta = \frac{B}{H}$$

$$\cos \theta = \frac{A}{H}$$

$$\tan \theta = \frac{P}{B}$$

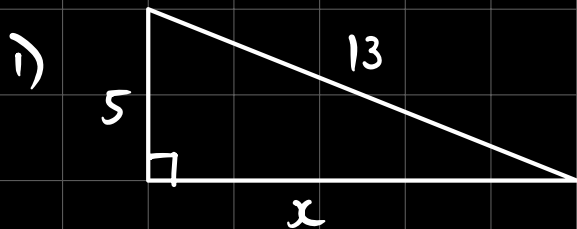
$$\tan \theta = \frac{O}{A}$$

Some ppl have
Curly Brown Hair

Through proper brushing.

SOH
CAH
TOA

Q. Find value of x



$$13^2 = x^2 + 5^2$$

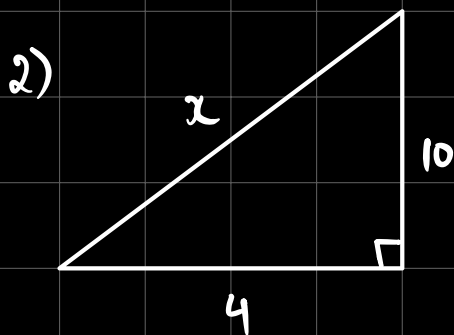
$$169 = x^2 + 25$$

$$169 - 25 = x^2$$

$$x^2 = 144$$

$$x = \sqrt{144}$$

$$x = 12$$



$$x^2 = 10^2 + 4^2$$

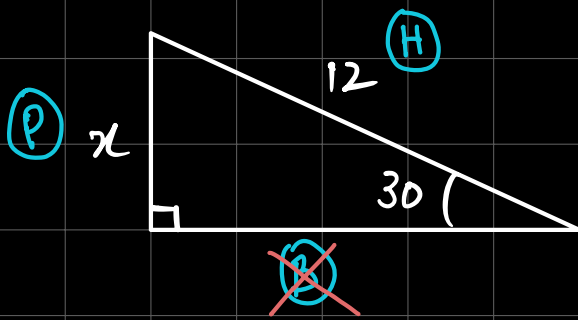
$$x^2 = 100 + 16$$

$$x^2 = 116$$

$$x = \sqrt{116}$$

$$x = 10.7703$$

3)



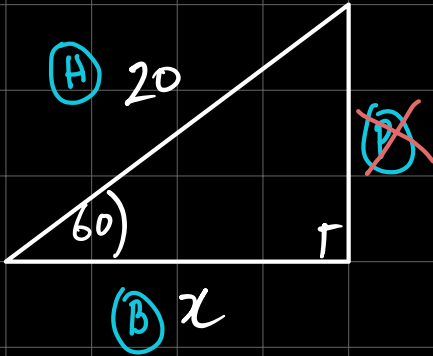
$$\sin \theta = \frac{P}{H}$$

$$\sin 30 = \frac{x}{12}$$

$$x = 12 \sin 30$$

$$x = 6$$

4)



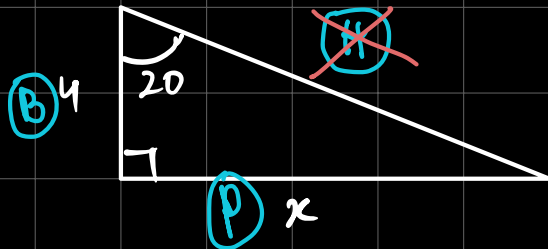
$$\cos \theta = \frac{B}{H}$$

$$\cos 60 = \frac{x}{20}$$

$$x = 20 \cos 60$$

$$x = 10$$

5)



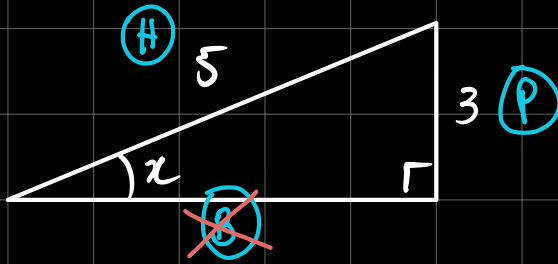
$$\tan \theta = \frac{P}{B}$$

$$\tan 20 = \frac{x}{4}$$

$$x = 4 \tan 20$$

$$x = 1.45588$$

6)



$$\sin \theta = \frac{P}{H}$$

$$\sin x = \frac{3}{5}$$

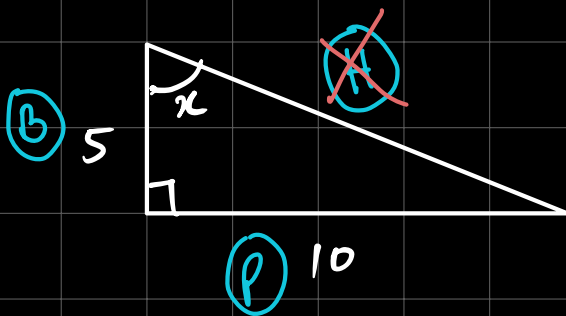
$$x = \sin^{-1}\left(\frac{3}{5}\right)$$

$$x = 36.8699$$

NOTE: $\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$

Inverse trig ratios are ALWAYS used when we have to find angle.

6)



$$\tan x = \frac{10}{5}$$

$$x = \tan^{-1}\left(\frac{10}{5}\right)$$

$$x = 63.43494$$

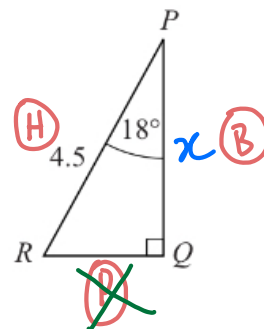
- 6 (a) In triangle PQR , $\angle RPQ = 18^\circ$, $\angle PQR = 90^\circ$ and $PR = 4.5$ m.

Find PQ .

$$\cos 18 = \frac{PQ}{4.5}$$

$$PQ = 4.5 \cos 18$$

$$PQ = 4.279754$$



Answer m [2]

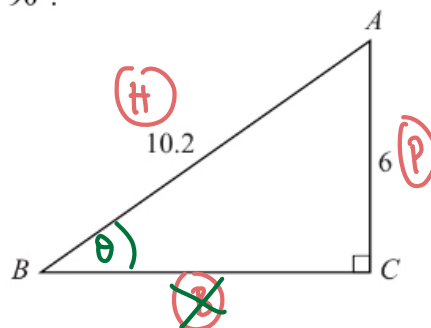
- (b) In triangle ABC , $AB = 10.2$ m, $AC = 6$ m and $\angle ACB = 90^\circ$.

(i) Find $\angle ABC$

$$\sin \theta = \frac{6}{10.2}$$

$$\theta = \sin^{-1}\left(\frac{6}{10.2}\right)$$

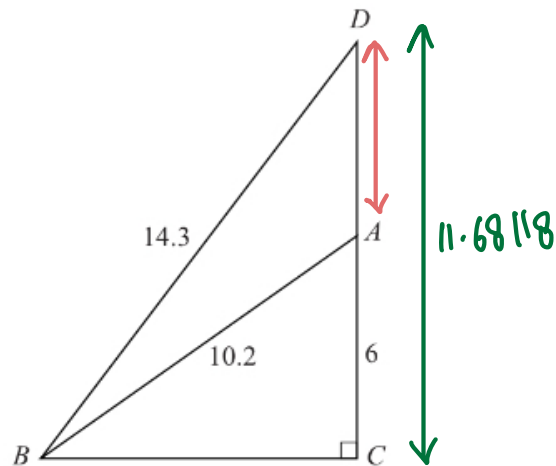
$$\theta = 36.0319$$



Answer [2]

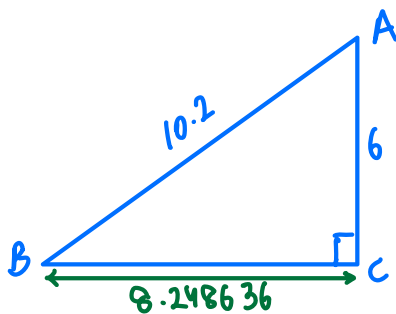
(ii)

$$AD = 11.68118 - 6 = 5.68118$$



D is the point on CA produced such that $BD = 14.3$ m.

Find AD.



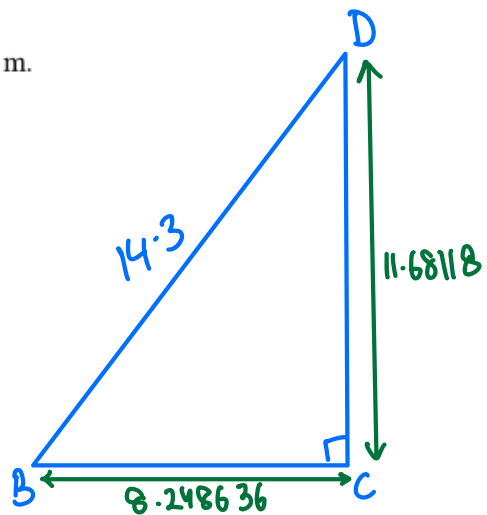
$$10.2^2 = CB^2 + 6^2$$

$$104.04 = CB^2 + 36$$

$$CB^2 = 68.04$$

$$CB = \sqrt{68.04}$$

$$CB = 8.248636$$



$$14.3^2 = 8.248636^2 + CD^2$$

$$204.49 = 68.04 + CD^2$$

$$CD^2 = 136.45$$

$$CD = \sqrt{136.45}$$

$$CD = 11.68118$$

Answer 5.68118 m [4]

ms

5.68

5.68118 → 5.68 ✓

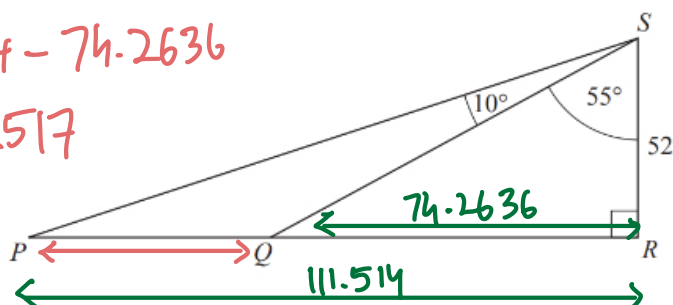
5.68075 → 5.68 ✓

5.680325 → 5.68 ✓

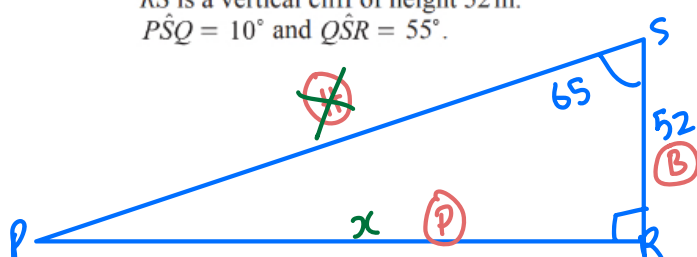
(b)

$$PQ = 111.514 - 74.2636$$

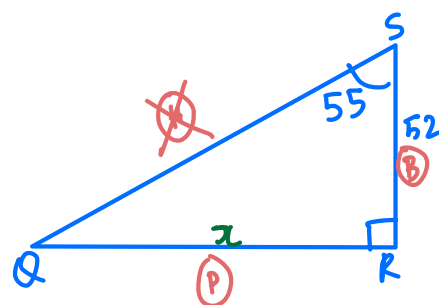
$$= 37.2517$$



Two boats are at the points P and Q .
 RS is a vertical cliff of height 52 m.
 $\angle PSQ = 10^\circ$ and $\angle QSR = 55^\circ$.



(ii) Calculate the distance, PQ , between the boats.



$$\tan 65 = \frac{PR}{52}$$

$$PR = 52 \tan 65$$

$$PR = 111.514$$

$$\tan 55 = \frac{QR}{52}$$

$$QR = 52 \tan 55$$

$$QR = 74.2636$$

Answerm [3]

4024/21/O/N/13

IF NOT RIGHT ANGLED TRIANGLE

Sine Rule



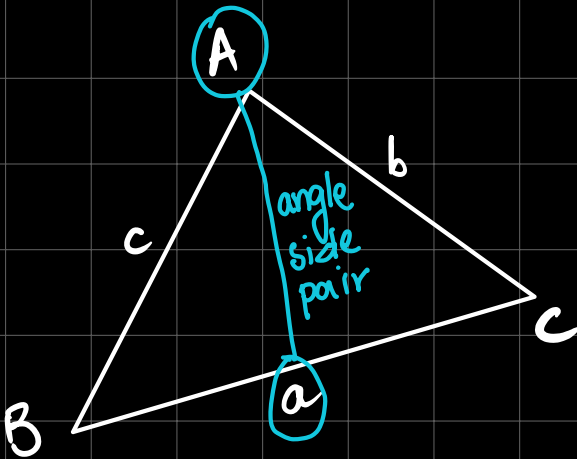
Sine Rule only works if one complete "angle-side pair" is given in question.

Cos Rule.



If no complete pair is given in question

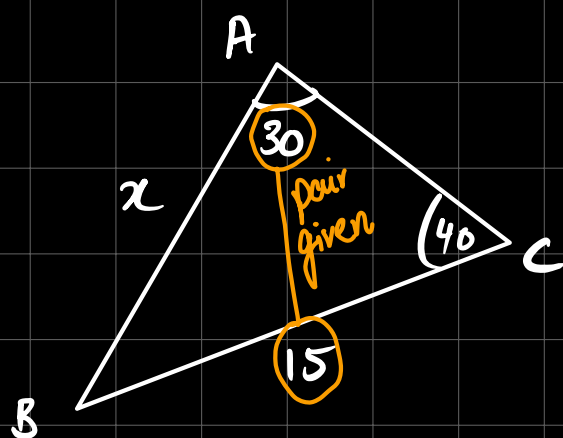
SINE RULE



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Sine Rule can only be used if one "angle-side pair" is given in the question.

Q.



SINE RULE

$$\frac{\sin 30}{15} = \frac{\sin 40}{x} \quad \left. \vphantom{\frac{\sin 30}{15} = \frac{\sin 40}{x}} \right\} M1$$

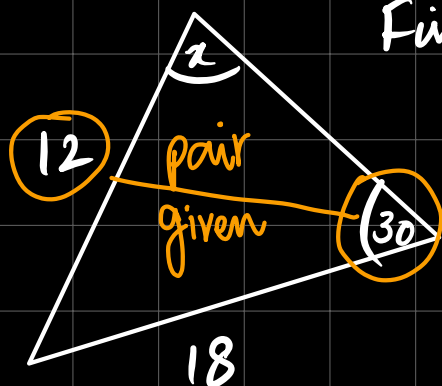
$$x \sin 30 = 15 \sin 40 \quad \left. \vphantom{x \sin 30 = 15 \sin 40} \right\} M1$$

$$x = \frac{15 \sin 40}{\sin 30}$$

$$x = 19.2836$$

$\left\{ \begin{array}{l} \sin^{-1} () \\ \cos^{-1} () \\ \tan^{-1} () \end{array} \right\}$ is only used if we are trying to find angle.

Q.



Find x.

SINE RULE

$$\frac{\sin 30}{12} = \frac{\sin x}{18}$$

$$12 \sin x = 18 \sin 30$$

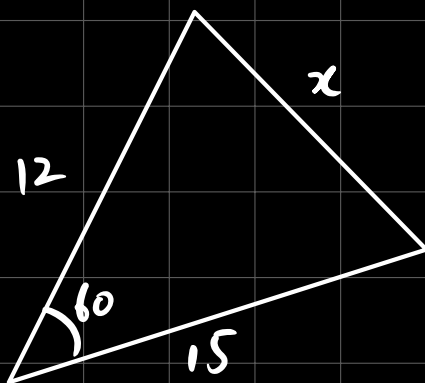
$$\sin x = \frac{18 \sin 30}{12}$$

$$\sin x = 0.75$$

$$x = \sin^{-1}(0.75)$$

$$x = 48.590377$$

Q.



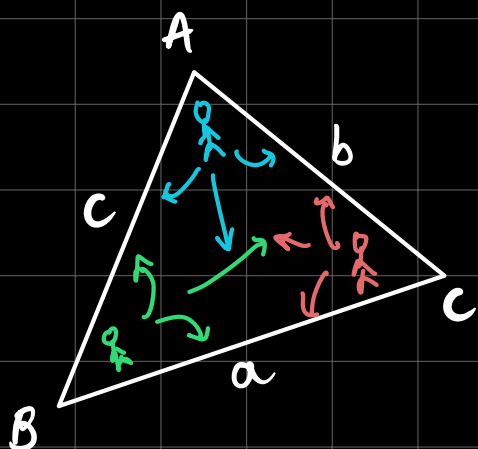
Not Right $\rightarrow$ Pythagoras $\times$
Trig Ratios $\times$

Complete Pair $\rightarrow x \rightarrow$ Sine $\times$
Rule

Cos Rule $\checkmark$



Cos Rule : In Cos Rule only one angle gets involved.



$$\cos A = \frac{\overbrace{b^2 + c^2}^{\text{ADD SQUARE OF NEIGHBORING SIDES}} - \overbrace{a^2}^{\text{SUB SQUARE OF OPPOSITE SIDE}}}{\underbrace{2bc}_{\text{neighboring sides}}}$$

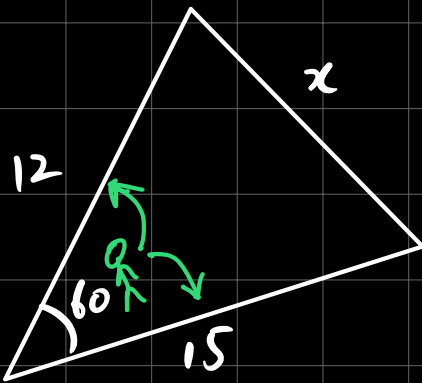
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

This is a 4 Mark part. Make sure you do all steps exactly like this.

use calculator or highlighted parts only.

Q.



Note: Jo cheese subject banani ho, it should be +ve.

$$\cos 60 = \frac{12^2 + 15^2 - x^2}{2(12)(15)}$$

$$\cos 60 = \frac{369 - x^2}{360}$$

$$360 \cos 60 = 369 - x^2$$

$$x^2 + 360 \cos 60 = 369$$

$$x^2 = 369 - 360 \cos 60$$

$$x^2 = 189$$

$$x = \sqrt{189}$$

$$x = 13.7477$$

2 Mark

M1 (Method)

✓

✓

✗

A1 (Answer)

✓

✗

✓

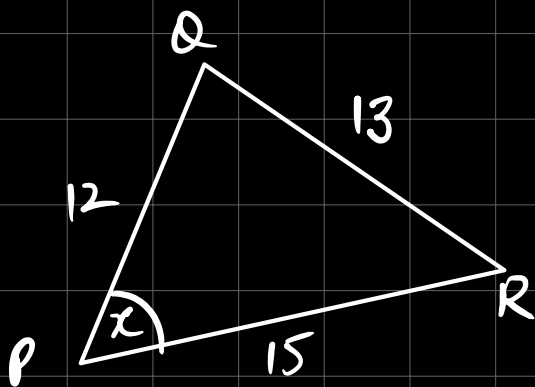
2

1

0

If a student has not earned M marks, you cannot earn A marks.

Q.



TRIGONOMETRY QUESTION

RIGHT ANGLED

angles
not involved

Angles
involved

PYTHAGORAS

2 marks

TRIG RATIOS

2 marks

NOT RIGHT
ANGLED TRIANGLE

One angle - side
pair is given

No pair
Given

SINE RULE

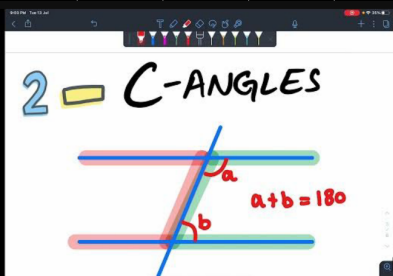
3 Marks

cos Rule

3/4
marks

Branches :

- 1) Area of a triangle
- 2) Angle of elevation / depression.
- 3) Bearings
- 4) How to use Trig in 3D shapes?



ANGLES | ANGLE PROPERTIES | O LEVELS MATHS | IGCSE MATHS

22K views · 2 years ago



ZAINEMATICS

AREA OF A TRIANGLE

TRIG.

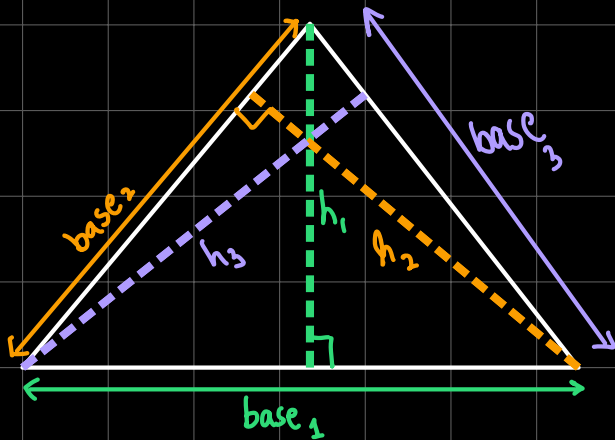
MENSURATION

20

20

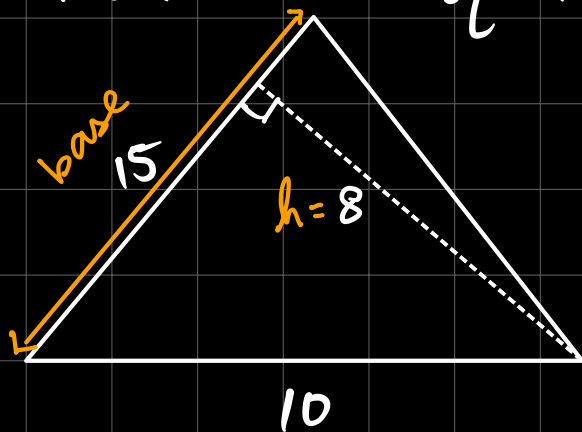
$$A = \frac{1}{2} \times b \times h$$

h = 90° DISTANCE FROM
THIRD POINT ON BASE LINE

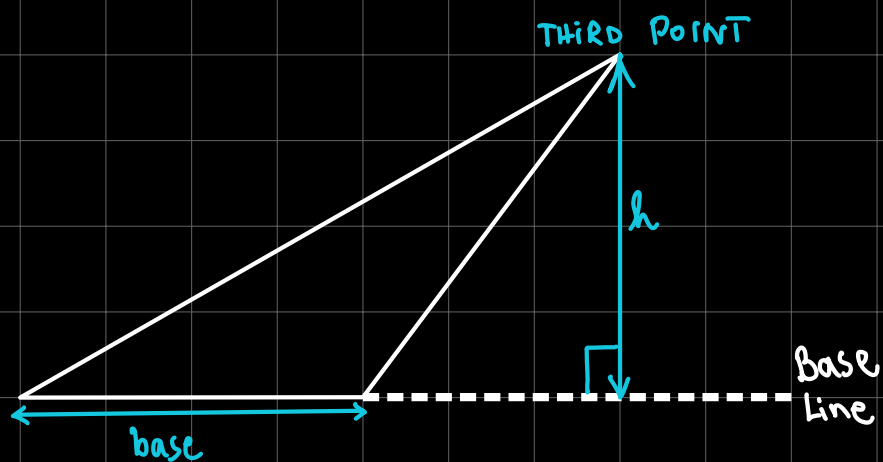


Rule: Every Triangle has 3 bases.
Each base will have its own height.
In formula you have to use
correct pair of base and height.

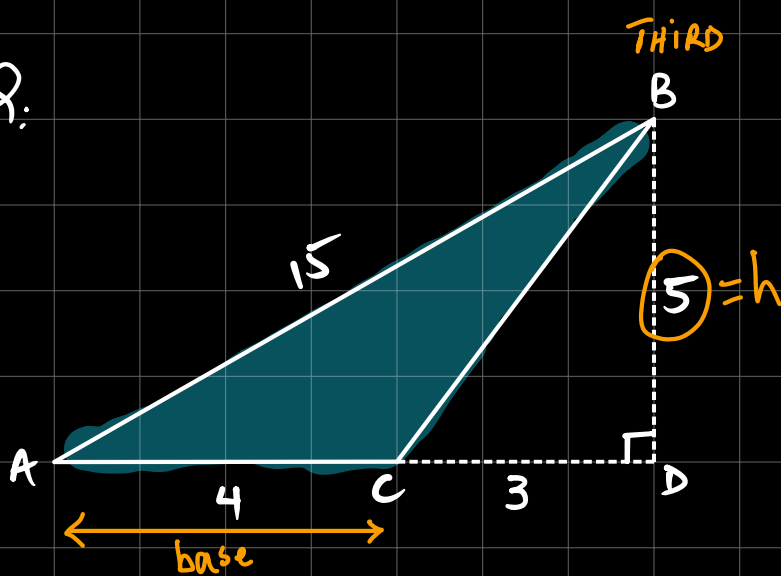
Q: Find area of this triangle:



$$\begin{aligned} A &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 15 \times 8 \\ &= 60 \end{aligned}$$



Q:

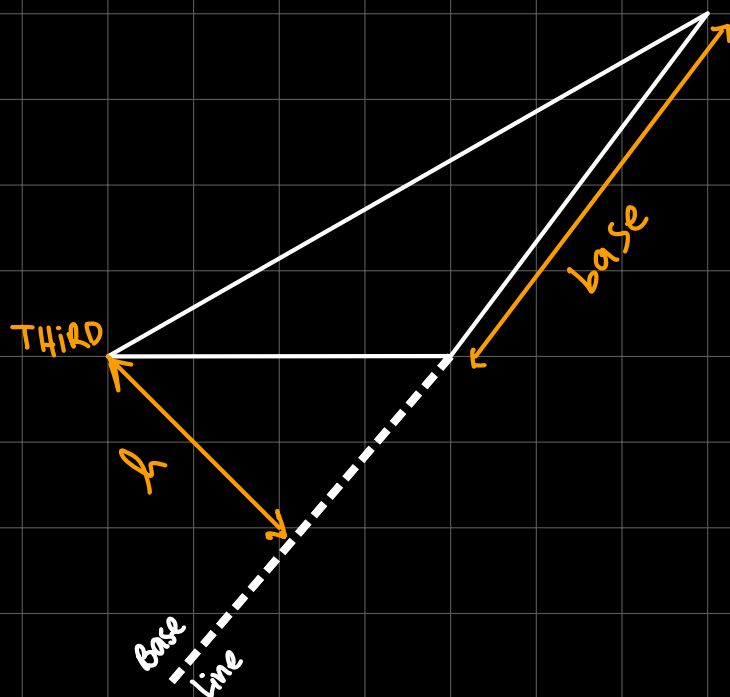
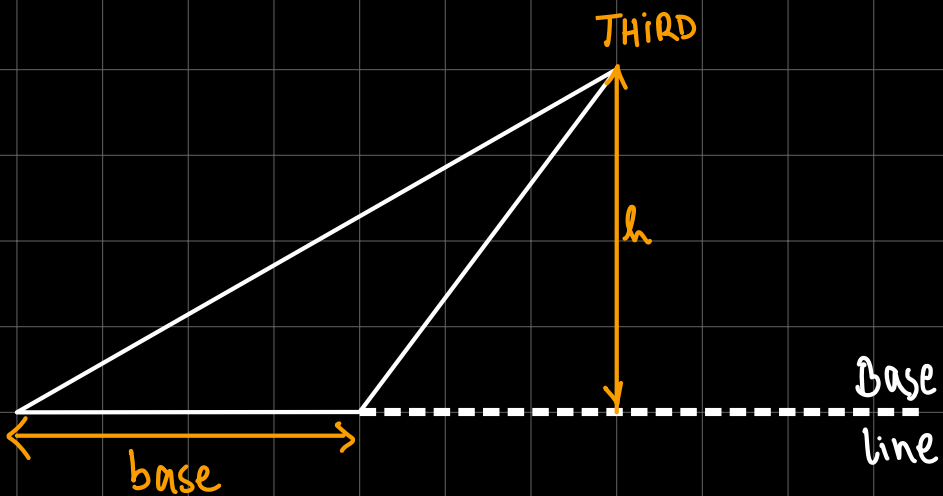
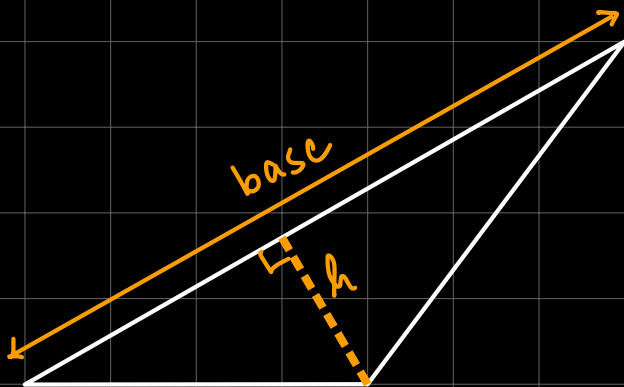


Find area of $\triangle ABC$

$$A = \frac{1}{2} \times b \times h$$

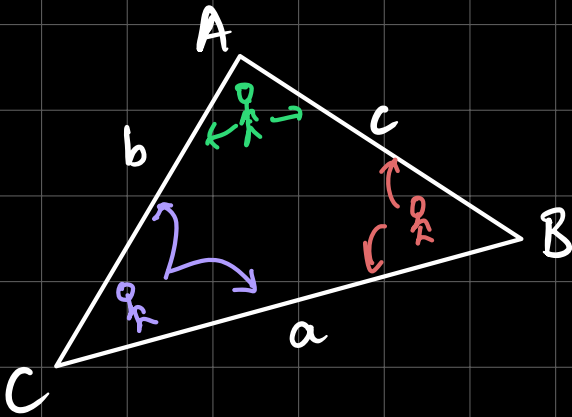
$$= \frac{1}{2} \times (4) \times (5)$$

$$A = 10$$



IF NO CORRECT PAIR OF BASE & HEIGHT
IS AVAILABLE???

Now The Question will give an angle



$$A = \frac{1}{2} \boxed{} \boxed{} \sin \bigcirc$$

neighboring sides.

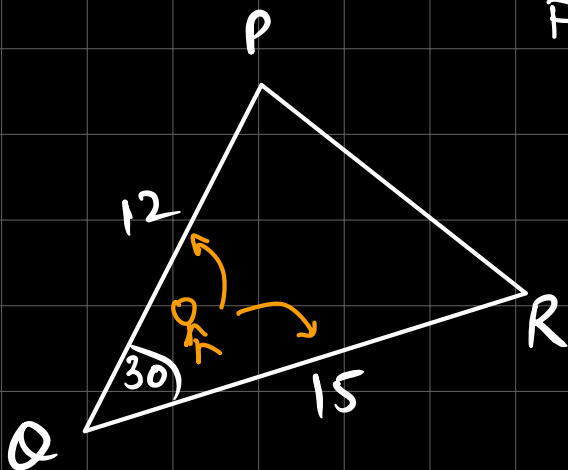
Angle

$$A = \frac{1}{2} \boxed{b} \boxed{c} \sin \bigcirc A$$

$$A = \frac{1}{2} \boxed{a} \boxed{c} \sin \bigcirc B$$

$$A = \frac{1}{2} \boxed{a} \boxed{b} \sin \bigcirc C$$

Q.



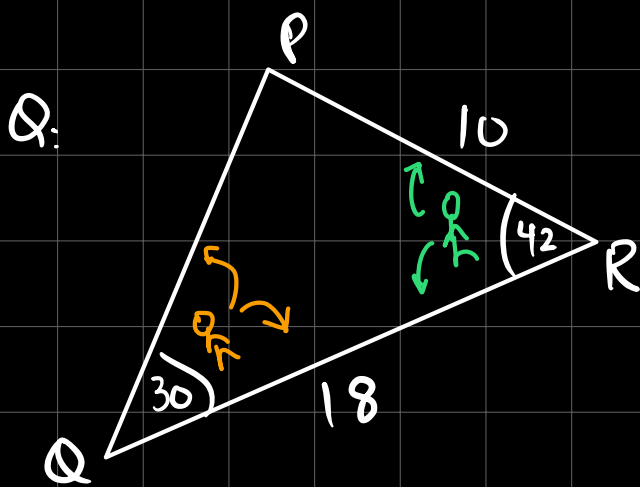
Find area of triangle PQR

$$A = \frac{1}{2} \boxed{12} \boxed{15} \sin \bigcirc 30$$

neighboring

Angle

$$A = 45$$



$$A = \frac{1}{2} \boxed{?} \boxed{18} \sin(30)$$

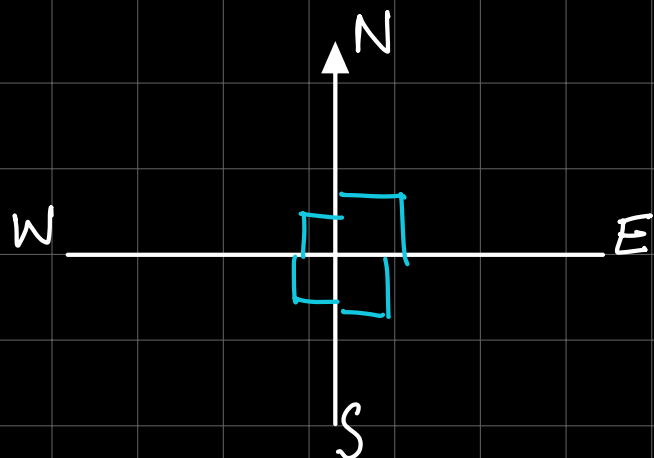
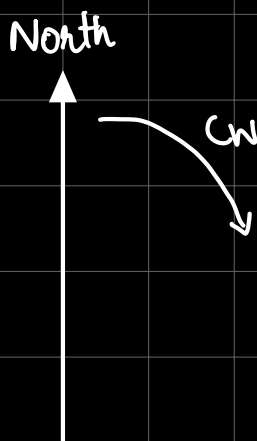
The other neighboring side is missing.

$$A = \frac{1}{2} \boxed{18} \boxed{10} \sin(42)$$

$$A = 60.22175$$

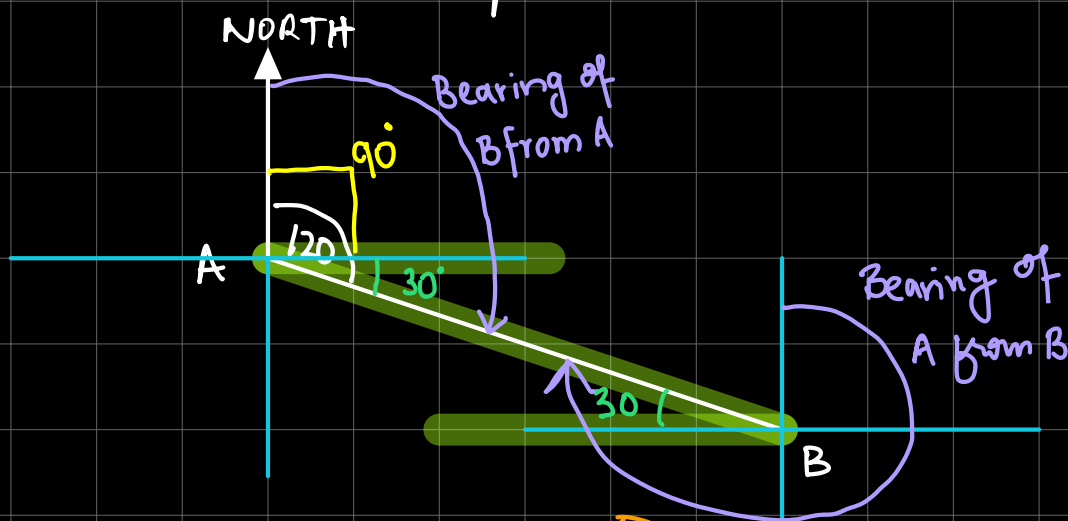
BEARINGS

Used to tell direction. in navy and aviation



we will do one easy question to understand the process.

Q.



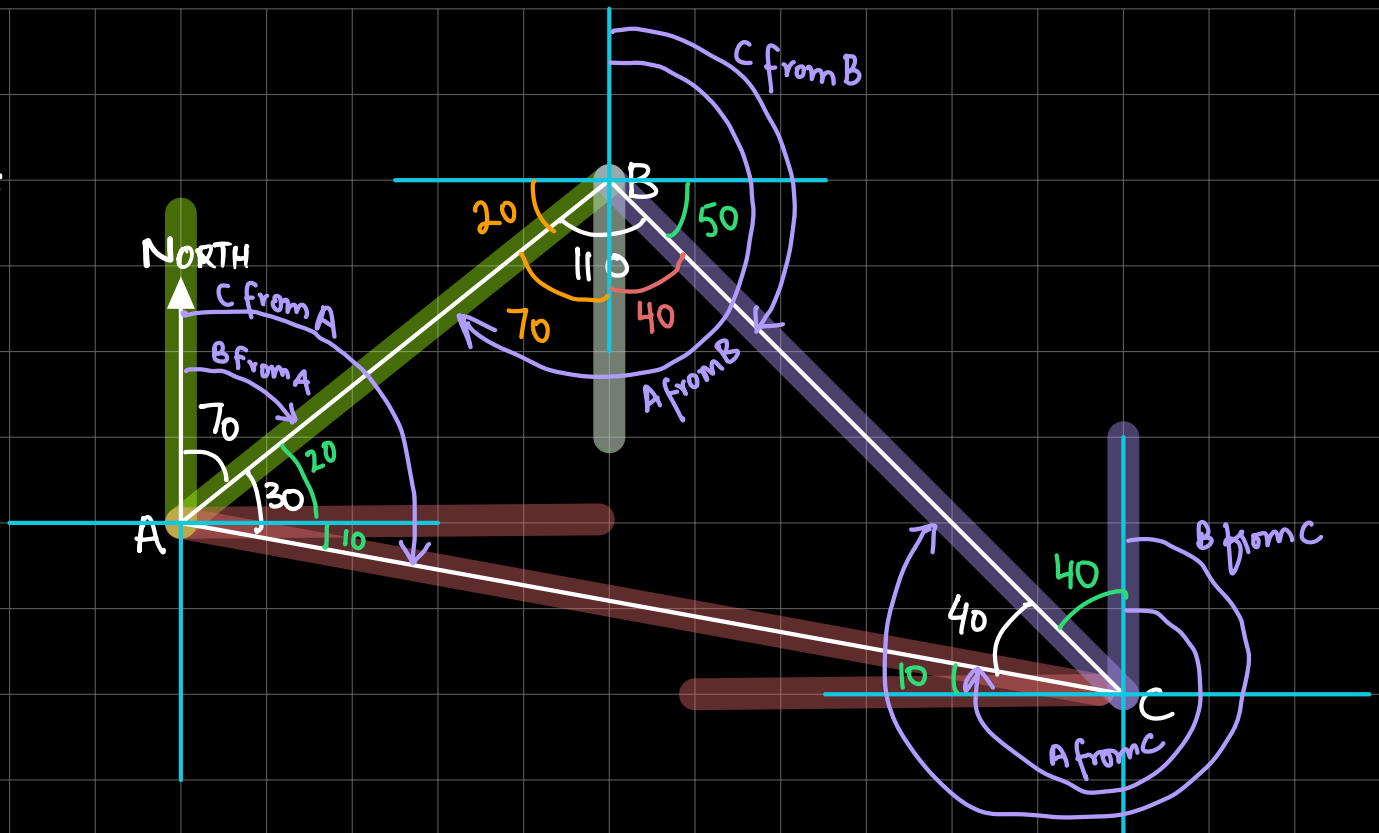
(a) Find bearing of B from A = 120

(b) Find bearing of A from B = $90 + 90 + 90 + 30 = 300$

PROCESS:

- 1- Make cross on alphabet after the word "from".
- 2- Split angles if possible
- 3- Mark all Z-angles
- 4- Find bearings.

Q:



Find Bearing of

(a) B from A = 070° *

(b) C from A = $70 + 30 = 100$

(c) C from B = $90 + 50 = 140$

(d) A from B = $90 + 90 + 70 = 250$

(e) A from C = $90 + 90 + 90 + 10 = 280$

(f) B from C = $90 + 90 + 90 + 10 + 40 = 360 - 40 = 320$

* $12.34^\circ \rightarrow 123.4^\circ$

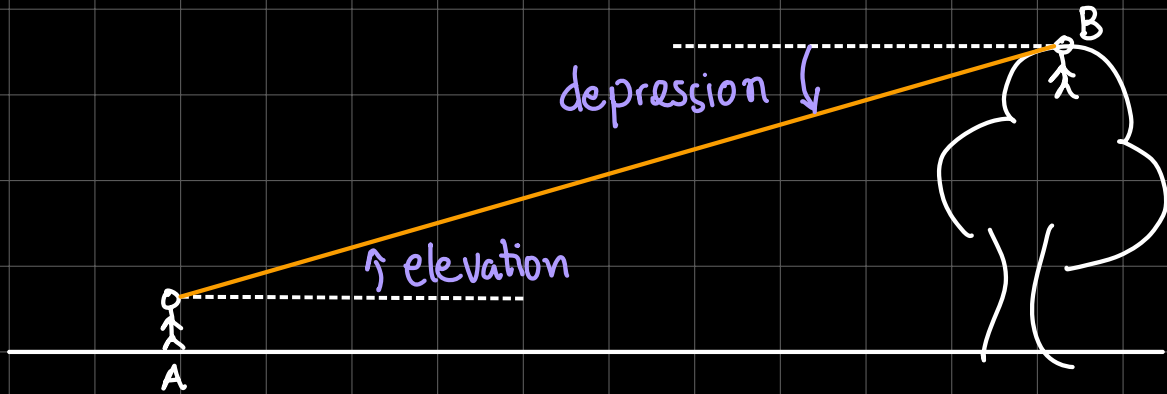
Rule: Must be Three digits before decimal.

$12.34 \rightarrow 012.34$

$6.4 \rightarrow 006.4$

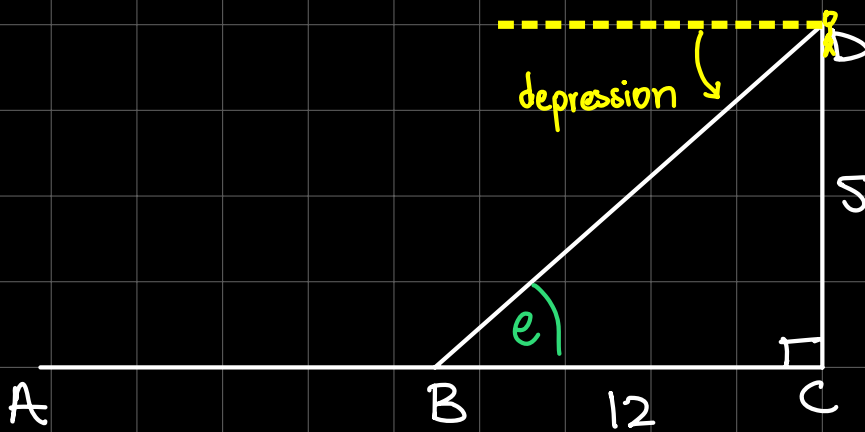
$0.5 \rightarrow 000.5$

ANGLE OF ELEVATION AND DEPRESSION



Note: 1) angle of elevation = angle of Depression. (Z-angles)

2) Both angle of elevation and angle of depression are made with a HORIZONTAL line.



Find angle of depression of B from D

$$\tan e = \frac{5}{12}$$

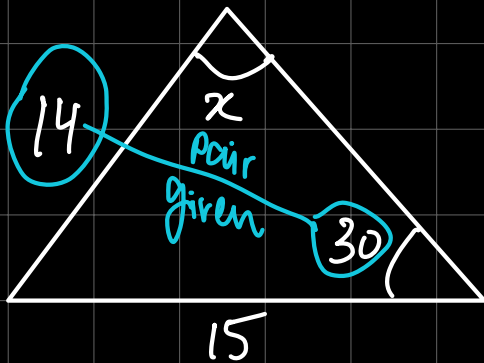
$$e = \tan^{-1}\left(\frac{5}{12}\right)$$

$$e = 22.61986$$

$$e = \text{Angle of depression} = 22.61986$$

(P2)

OBTUSE ANGLE VARIATION



Q: Given that x is obtuse, Find x .

STEP 1: IGNORE OBTUSE, FIND x .

$$\frac{\sin 30}{14} = \frac{\sin x}{15}$$

$$14 \sin x = 15 \sin 30$$

$$\sin x = \frac{15 \sin 30}{14}$$

$$\sin x = \frac{15}{28}$$

$$x = \sin^{-1}\left(\frac{15}{28}\right)$$

$$x = 32.3924' \text{ (ACUTE)}$$

STEP 2: NOW SUBTRACT FROM 180 TO GET OBTUSE

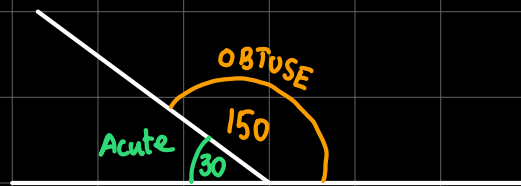
$$\text{OBTUSE} = 180 - \text{ACUTE}$$

$$= 180 - 32.3924$$

$$= 147.6076$$

0 - 90	Acute
90	Right
90 - 180	Obtuse
180	Straight
180 - 360	Reflex

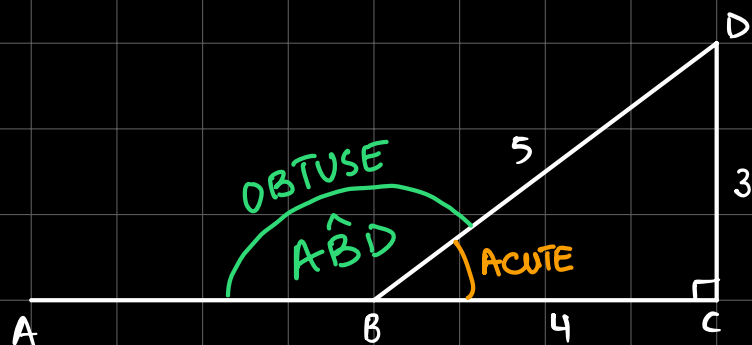
P1 SPECIAL CONCEPT (1 MARK)



$$\begin{aligned} \sin 30 &= 0.5 & \sin 150 &= 0.5 \\ \cos 30 &= 0.866 & \cos 150 &= -0.866 \\ \tan 30 &= 0.577 & \tan 150 &= -0.577 \end{aligned}$$

when there are two neighboring angles on a straight line, one is acute other is obtuse. Numerical value of sin/cos/tan of both angles is exactly same.

cos and tan of obtuse angle is negative.



Find value of:

$$(i) \sin \hat{C} B D = \frac{P}{H} = \frac{3}{5}$$

$$(ii) \cos \hat{C} B D = \frac{H}{B} = \frac{4}{5}$$

$$(iii) \tan \hat{C} B D = \frac{P}{B} = \frac{3}{4}$$

$$(iv) \cos \underbrace{\hat{A} B D}_{\text{OBTUSE}} = -\frac{4}{5}$$

$$\cos(\text{Acute}) = \frac{B}{H} = \frac{4}{5}$$

$$\cos(\text{obtuse}) = -\frac{4}{5}$$

$$(v) \tan \hat{A} B D = -\frac{3}{4}$$

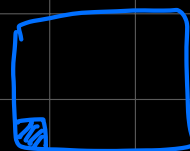
$$\tan(\text{Acute}) = \frac{P}{B} = \frac{3}{4}$$

$$\tan(\text{obtuse}) = -\frac{3}{4}$$

$$(vi) \sin \underbrace{\hat{A} B D}_{\text{OBTUSE}} = \frac{3}{5}$$

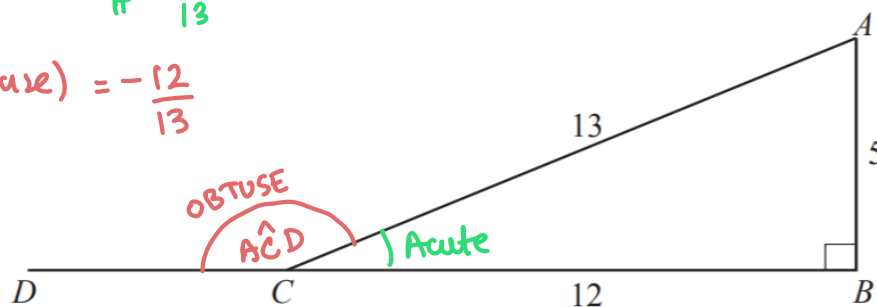
$$\sin(\text{Acute}) = \frac{P}{H} = \frac{3}{5}$$

$$\sin(\text{obtuse}) = \frac{3}{5}$$



(b) $\cos(\text{Acute}) = \frac{B}{H} = \frac{12}{13}$

$\cos(\text{obtuse}) = -\frac{12}{13}$



~~$\sin(180-\theta) = \sin \theta$
 $\cos(180-\theta) = -\cos \theta$
 $\tan(180-\theta) = -\tan \theta$~~

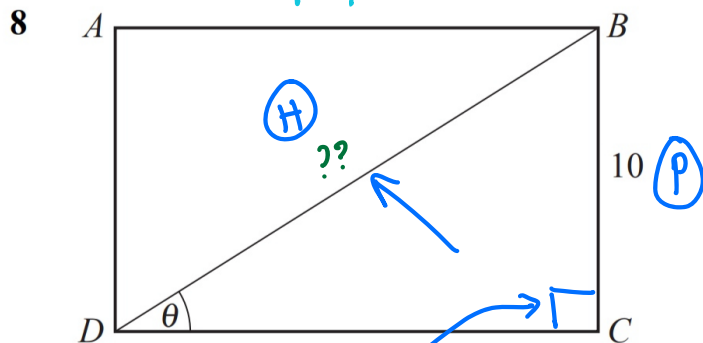
Triangle ABC is right-angled at B and BC is produced to D .

$AB = 5$ cm, $BC = 12$ cm and $AC = 13$ cm.

Write down the value of $\cos \hat{ACD}$.

Answer $\cos \hat{ACD} = \dots\dots\dots -\frac{12}{13} \dots\dots\dots [1]$

Treat this as if you have calculator:



$\sin \theta$	$\frac{5}{13}$
$\cos \theta$	$\frac{12}{13}$
$\tan \theta$	$\frac{5}{12}$

$ABCD$ is a rectangle with $BC = 10$ cm.

Using as much information from the table as is necessary, calculate BD .

$\sin \theta = \frac{10}{BD}$
 $\frac{5}{13} = \frac{10}{BD}$

$5 BD = 10 \times 13$
 $BD = \frac{10 \times 13}{5}$
 $BD = 26$

Answer $BD = \dots\dots\dots 26 \dots\dots\dots$ cm [2]

Right $\left\{ \begin{array}{l} \text{Pyth} \\ \text{Sin/cos/tan} \end{array} \right.$

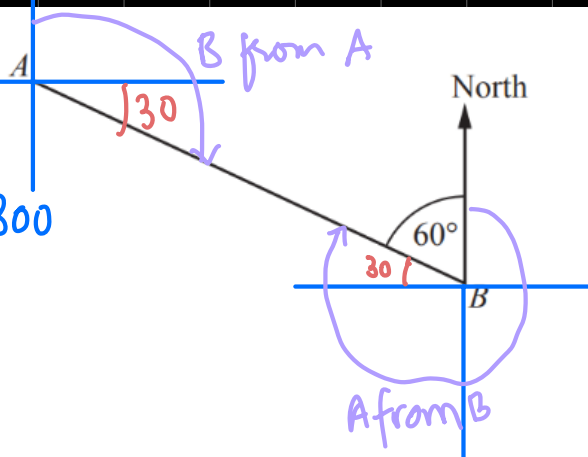
Non Right $\left\{ \begin{array}{l} \text{Sine Rule} \\ \text{cos Rule} \end{array} \right.$

- 4 The diagram shows the positions of A and B .

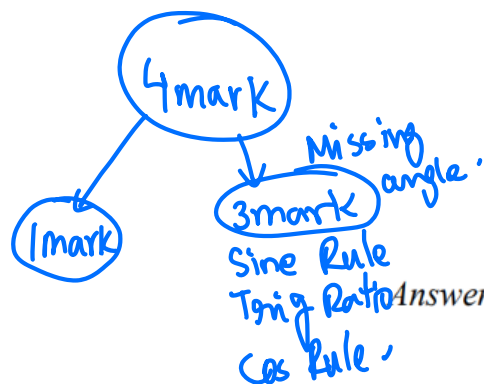
Find the bearing of

(a) A from B , $= 90 + 90 + 90 + 30 = 360 - 60 = 300$

(b) B from A . $= 90 + 30 = 120$



Bearing ??
(1 mark)



Answer (a) [1]

(b) [1]

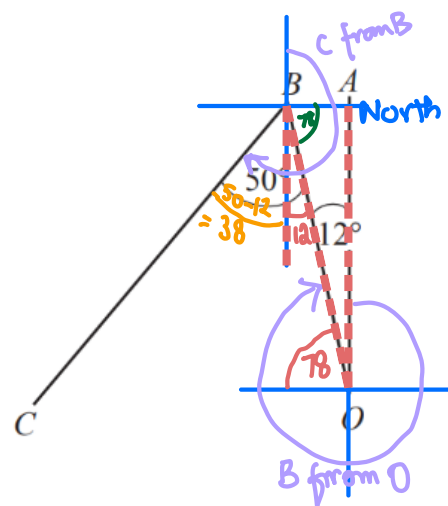
- 5 A is due North of O .

- (a) A ship sailed from O to B , where $\angle AOB = 12^\circ$.
Write down the bearing of B from O . $= 360 - 12 = 348$

- (b) At B , the ship turned and sailed to C , where $\angle OBC = 50^\circ$.
Calculate the bearing of C from B .

$$90 + 90 + 38$$

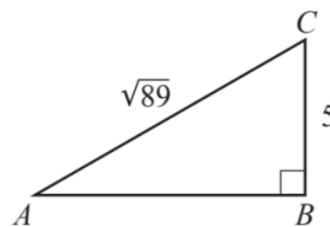
$$= 218$$



Answer (a) 348 [1]

(b) 218 [1]

- 13 In the diagram, $\hat{ABC} = 90^\circ$,
 $BC = 5 \text{ cm}$ and $AC = \sqrt{89} \text{ cm}$.



- (a) What special kind of number is $\sqrt{89}$?

Answer (a) Irrational [1]

- (b) Calculate AB .

Pythagoras

$$(\sqrt{89})^2 = 5^2 + AB^2$$

$$89 = 25 + AB^2$$

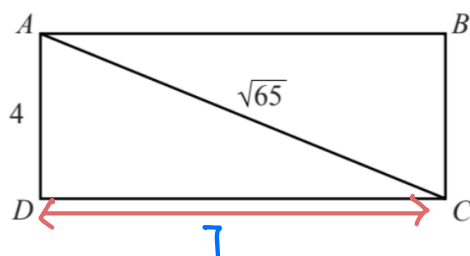
$$AB^2 = 89 - 25$$

$$AB^2 = 64$$

$$AB = \sqrt{64} = 8$$

Answer (b) 8 cm [2]

17



$ABCD$ is a rectangle with $AC = \sqrt{65} \text{ cm}$ and $AD = 4 \text{ cm}$.

Calculate the area of $ABCD$.

Pythagoras:

$$(\sqrt{65})^2 = 4^2 + CD^2$$

$$65 = 16 + CD^2$$

$$CD^2 = 65 - 16$$

$$CD^2 = 49$$

$$CD = \sqrt{49} = 7$$

$$\begin{aligned} \text{Area} &= L \times w \\ &= 7 \times 4 \\ &= 28 \end{aligned}$$

Answer 28 cm² [3]

•
Congruency
Similarity
HCF/LCM
Symmetry
Polygons

